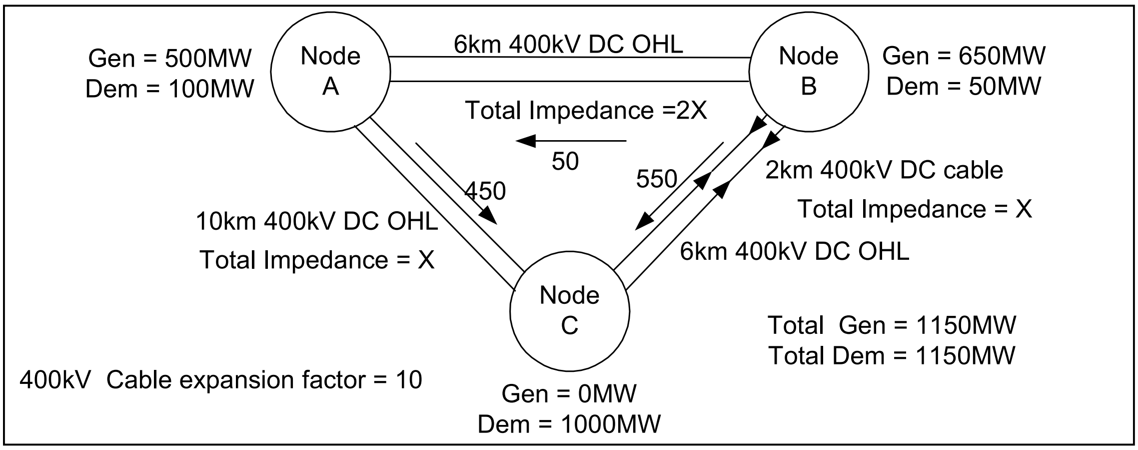
Seculf : A worked example

Take the following simple 3- node network



	Impedance (X)	Susceptance (Y)	Equivalent length
Branch A-B	**2**	**0.5**	**6.0**
Branch B-C	**1**	**1.0**	**26.0**
Branch A-C	**1**	**1.0**	**10.0**

1. Calculate Intact Nodal marginal Cost

Create a Susceptance Matrix and invert it to calculate sensitivity coefficients of branch power flows, (see Appendix for matrix theory).

Note: this 3x3 matrix is singular and cannot be inverted without removing an associated row and column. The row and column associated with the reference node (shown by shading) is chosen, as the nodal marginal cost for the reference node is zero.

	A	**B**	**C**
A	1.50	-0.50	-1.00
B	-0.5	1.5	-1.00
C	-1.00	-1.00	2.00

Inverting the 2x2 matrix above;

	B	**C**
B	1.0	0.5
C	0.5	0.75

The sensitivity coefficients may be calculated to show the marginal effect of each node:

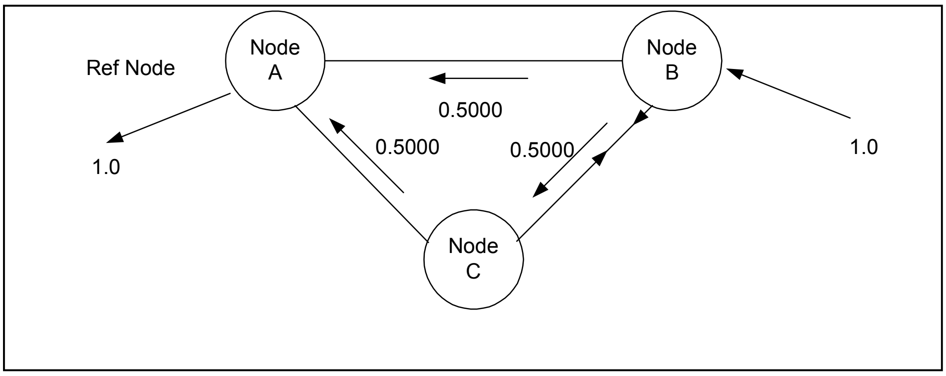
Node B

Coefficient of branch B-A = $(Y^{-1}_{(B,B)} - Y^{-1}_{(A,B)})^{*}\ Y_{branch\ A\text{-}B}$ = (1.0-0.0)* 0.5 = 0.5

Coefficient of branch B-C = $(Y^{-1}_{(B,B)} - Y^{-1}_{(C,B)})^{*}\ Y_{branch\ B\text{-}C}$ = (1.0-0.5)* 1.0 = 0.5

Coefficient of branch A-C = $(Y^{-1}_{(A,B)} - Y^{-1}_{(C,B)})^{*}\ Y_{branch\ A\text{-}C}$ = (0.0-0.5)* 1.0 = -0.5

The coefficients may be shown diagrammatically as:



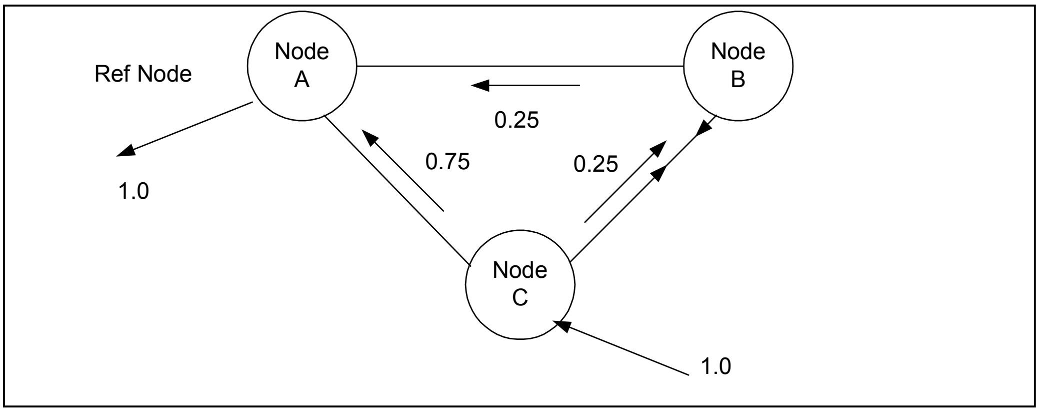
The Nodal Marginal effect (cost) in MWkm is given by the sum of the coefficients multiplied by the respective branch lengths

NMC for Node B
= (Coeff B-A *length B-A) + (Coeff B-C * length B-C) + (Coeff A-C * length A-C)
= (0.5*6.0) + (0.5*26.0) + (-0.5*10.0)
= 11 MWkm

Node C

Coefficient of branch B-A = $(Y^{-1}_{(B,C)} - Y^{-1}_{(A,C)})* Y_{branch\ AB}$ = (0.5-0.0)* 0.5 = 0.25
Coefficient of branch B-C = $(Y^{-1}_{(B,C)} - Y^{-1}_{(C,C)})* Y_{branch\ BC}$ = (0.5-0.75)* 1.0 = -0.25
Coefficient of branch A-C = $(Y^{-1}_{(A,C)} - Y^{-1}_{(C,C)})* Y_{branch\ AC}$ = (0.0-0.75)* 1.0 = -0.75

The coefficients may be shown diagrammatically as:



Similarly the NMC for node C
= (Coeff B-A *length B-A) + (Coeff B-C * length B-C) + (Coeff A-C * length A-C)=
= (0.25*6.0) + (-0.25*26.0) + (-0.75*10.0)
= -12.5 MWkm

2. Calculate Secured Nodal marginal Cost

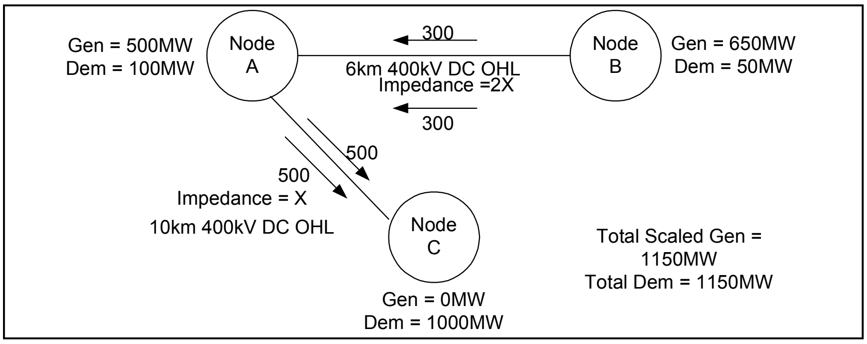
Summary of Branch flow for Intact and Contingency Cases

	Intact	A-B sc	A-B dc	B-C sc	B-C dc	A-C sc	A-C dc	Max Flow
Branch B-A 1	-25.0	0.0	0.0	-80.0	-300.0	20.0	200.0	300.0
Branch B-A 2	-25.0	-33.33	0.0	-80.0	-300.0	20.0	200.0	300.0
Branch B-C 1	275.0	283.33	300.0	0.0	0.0	320.0	500.0	500.0
Branch B-C 2	275.0	283.33	300.0	440.0	0.0	320.0	500.0	500.0
Branch A-C 1	225.0	216.67	200.0	280.0	500.0	0.0	0.0	500.0
Branch A-C 2	225.0	216.67	200.0	280.0	500.0	360.0	0.0	500.0

Therefore, only consider the effect of the contingencies that gives the maximum flow on the circuits, i.e. B-C dc and AC-dc

Contingency B-C

For contingency B-C dc, a new Susceptance matrix may be calculated & inverted:



	B	C
B	0.5	0.0
C	0.0	1.0

,
inverting;

	B	C
B	2.0	0.0
C	0.0	1.0

The sensitivity coefficients may be calculated in exactly the same way:

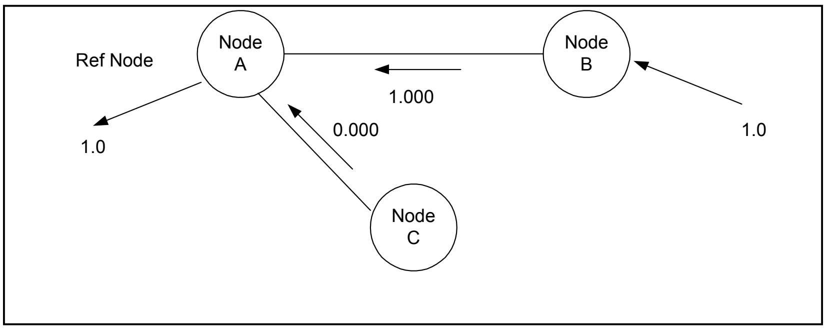
Node B

Coefficient of branch B-A = $(Y^{-1}_{(B,B)} - Y^{-1}_{(A,B)})^* Y_{branch\ AB}$ = (2.0-0.0)* 0.5 = 1.0

Coefficient of branch B-C = $(Y^{-1}_{(B,B)} - Y^{-1}_{(C,B)})^* Y_{branch\ BC}$ = (2.0-0.0)* 0.0 = 0.0

Coefficient of branch A-C = $(Y^{-1}_{(A,B)} - Y^{-1}_{(C,B)})^* Y_{branch\ AC}$ = (0.0-0.0)* 1.0 = 0.0

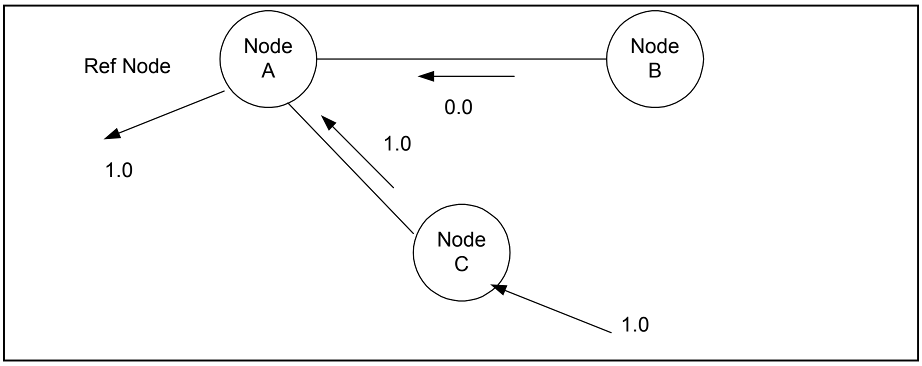
The coefficients may be shown diagrammatically as:



Node C

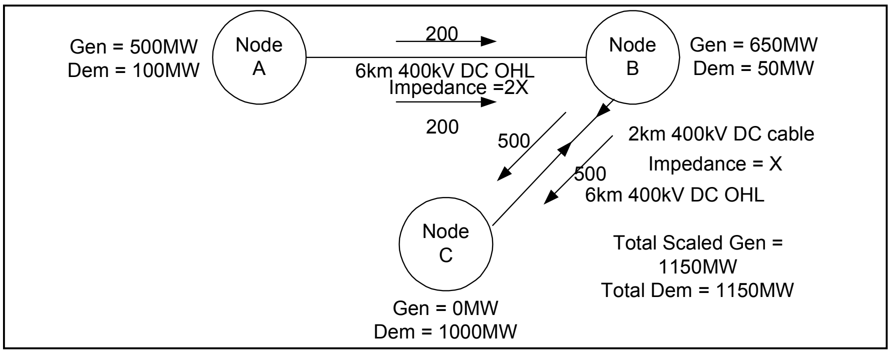
Coefficient of branch B-A = $(Y^{-1}_{(B,C)} - Y^{-1}_{(A,C)})* Y_{branch\ AB}$ = (0.0-0.0)* 0.5 = 0.0
Coefficient of branch B-C = $(Y^{-1}_{(B,C)} - Y^{-1}_{(C,C)})* Y_{branch\ BC}$ = (0.0-1.0)* 0.0 = 0.0
Coefficient of branch A-C = $(Y^{-1}_{(A,C)} - Y^{-1}_{(C,C)})* Y_{branch\ AC}$ = (0.0-1.0)* 1.0 = -1.0

The coefficients may be shown diagrammatically as:



Contingency A-C

For contingency A-C dc, a new Susceptance matrix may be calculated & inverted:



	B	C
B	1.5	-1.0
C	-1.0	1.0

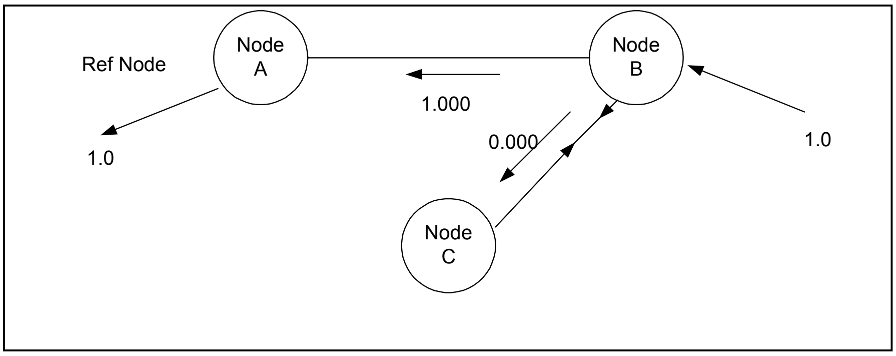
,
inverting gives,

	B	C
B	2.0	2.0
C	2.0	3.0

The sensitivity coefficients may be calculated as:

Node B
Coefficient of branch B-A = $(Y^{-1}_{(B,B)} - Y^{-1}_{(A,B)})* Y_{branch\ AB} = (2.0-0.0)* 0.5 = 1.0$
Coefficient of branch B-C = $(Y^{-1}_{(B,B)} - Y^{-1}_{(C,B)})* Y_{branch\ BC} = (2.0-2.0)* 1.0 = 0.0$
Coefficient of branch A-C = $(Y^{-1}_{(A,B)} - Y^{-1}_{(C,B)})* Y_{branch\ AC} = (0.0-2.0)* 0.0 = 0.0$

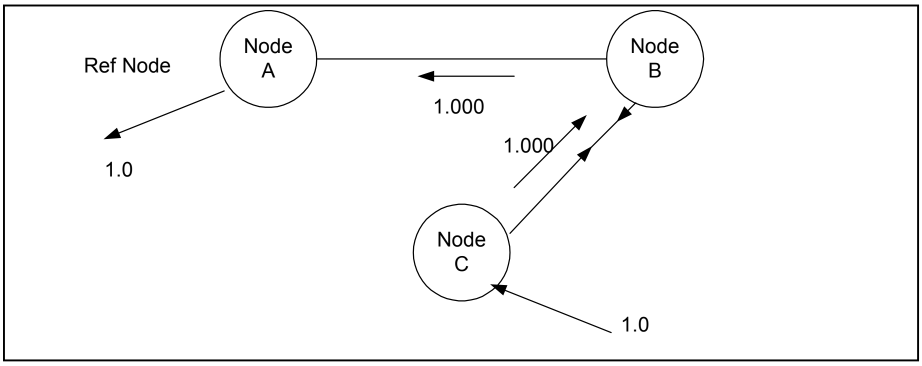
The coefficients may be shown diagrammatically as:



Node C

Coefficient of branch B-A = $(Y^{-1}_{(B,C)} - Y^{-1}_{(A,C)})* Y_{branch\ AB} = (2.0-0.0)* 0.5 = 1.0$
Coefficient of branch B-C = $(Y^{-1}_{(B,C)} - Y^{-1}_{(C,C)})* Y_{branch\ BC} = (2.0-3.0)* 1.0 = -1.0$
Coefficient of branch A-C = $(Y^{-1}_{(A,C)} - Y^{-1}_{(C,C)})* Y_{branch\ AC} = (0.0-3.0)* 0.0 = 0.0$

The coefficients may be shown diagrammatically as:



Secured Marginal costs

The secured marginal cost may be calculated using the branch sensitivity coefficients from the contingencies that caused the maximum flow in that branch, i.e. for branches A-B and A-C use coefficients from contingency B-C dc; for branch B-C use coefficients from contingency A-C dc.

For secured NMC Node B,

= (Coeff B-A * length B-A) + (Coeff A-C * length A-C) +(Coeff B-C * length B-C)

From Contingency B-C dc From Contingency A-C dc

= (1.0 * 6.0) + (0.0 * 10.0) + (0.0 * 26.0)

= 6.0 MWkm

For secured NMC Node C,

= (Coeff B-A * length B-A) + (Coeff A-C * length A-C) +(Coeff B-C * length B-C)

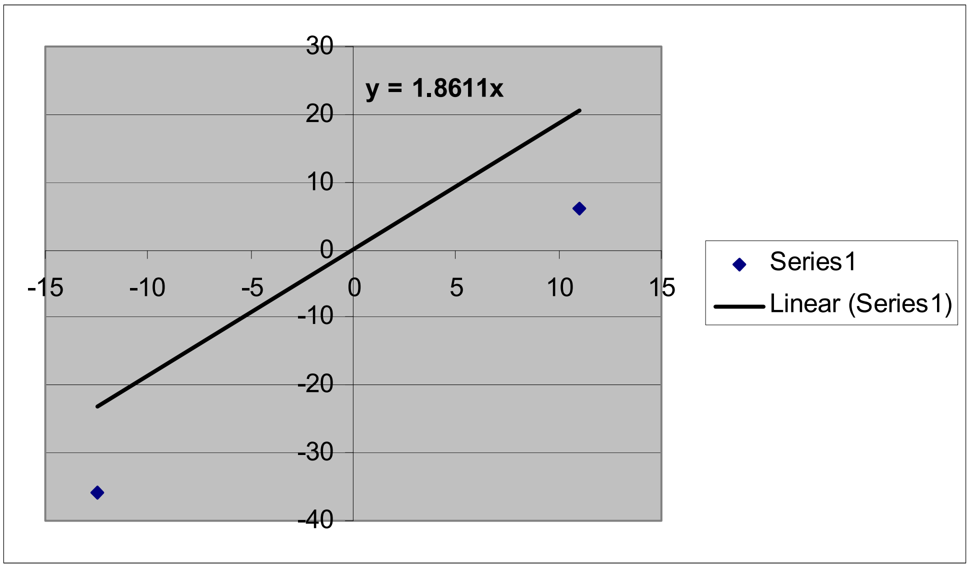
From Contingency B-C dc From Contingency A-C dc

= (0.0 * 6.0) + (-1.0 * 10.0) + (-1.0 * 26.0)

= -36.0 MWkm

3. Calculate Security Factor

	Intact	Secured
B	11	6
C	-12.5	-36



Appendix

Matrix Theory

For matrices there is no such thing as division, however they can be multiplied together. Using the reciprocal fraction or inverse of a number has the same effect as division, e.g. 18 divided by 3 is the same as 18 multiplied by 1/3.

The inverse of a matrix A is written as A^{-1}

A diagonal matrix whose non-zero entries are all 1's is called the *identity matrix*, in a similar way 1 is the "identity" in maths, e.g. 10 multiplied by 1 is 10. The Identity matrix is written as I.

For a 2x2 matrix the identity is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Note: A. I = A also, A^{-1}. I = A^{-1} and A. A^{-1} = I, e.g.

$$\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Not all square matrices have inverse matrix, if the *determinant* of a matrix is zero then it will not have an inverse and the matrix is said to be singular. Only non-singular matrices have an inverse.

Determinant of a 2x2 matrix
Assuming A is an arbitrary 2x2 matrix A, where the elements are given by:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

then the determinant of a this matrix is as follows:

$$\det(A) = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

Creating a Susceptance Matrix

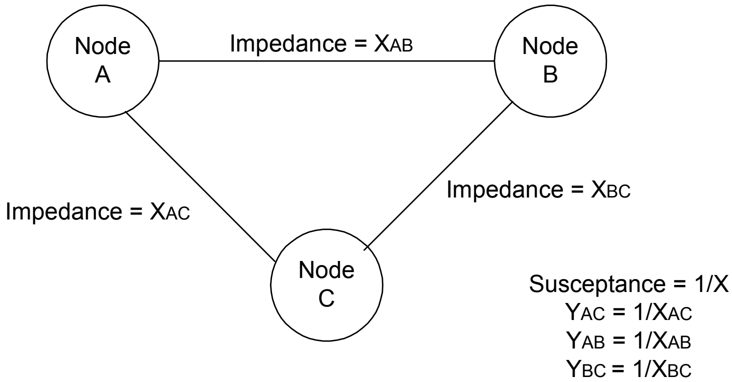
1. Network Theory

The Suceptance Matrix (or more commonly the Admittance Matrix in "Network Theory") may be defined as the Degree matrix minus the Adjacency matrix.

The Degree Matrix of a network is a diagonal only matrix which contains information about the degree of each node, i.e. for node i the degree is the sum of the branches incident to node i with loops counting twice.

The Adjacency Matrix of a network is where the non-diagonal entry e.g. Y_{ij} is the sum of branches directly connecting node i to node j. The diagonal entry e.g. Y_{ii} is the sum of loop branches connected to node i with loops counting twice.

Therefore, for the following three node example,



the Degree Matrix minus the Adjacency Matrix is given by:

$$\begin{bmatrix} Y_{AB}+Y_{AC} & 0 & 0 \\ 0 & Y_{BC}+Y_{AB} & 0 \\ 0 & 0 & Y_{AC}+Y_{BC} \end{bmatrix} - \begin{bmatrix} 0 & Y_{AB} & Y_{AC} \\ Y_{AB} & 0 & Y_{BC} \\ Y_{AC} & Y_{BC} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} Y_{AB}+Y_{AC} & -Y_{AB} & -Y_{AC} \\ -Y_{AB} & Y_{BC}+Y_{AB} & -Y_{BC} \\ -Y_{AC} & -Y_{BC} & Y_{AC}+Y_{BC} \end{bmatrix}$$

2. Application to DC Theory

Using the same three node example given above and the simplified DCLF equation of

$$P = theta/X$$

Then power flow equation may be created;

Power $Flow_{AB}$ = $(theta_A - theta_B) * Y_{AB}$

Power $Flow_{AC}$ = $(theta_A - theta_C) * Y_{AC}$

Power $Flow_{BC}$ = $(theta_B - theta_C) * Y_{BC}$

And;

P_A = Power $Flow_{AB}$ + Power $Flow_{AC}$
= $[(theta_A - theta_B) * Y_{AB}] + [(theta_A - theta_C) * Y_{AC}]$

P_B = -Power $Flow_{AB}$ + Power $Flow_{BC}$
= $[-(theta_A - theta_B) * Y_{AB}] + [(theta_B - theta_C) * Y_{BC}]$

P_C = -Power $Flow_{AC}$ - Power $Flow_{BC}$
= $[-(theta_A - theta_C) * Y_{AC}] - [(theta_B - theta_C) * Y_{BC}]$

Therefore, this can be represented as;

$$\begin{pmatrix} P_A \\ P_B \\ P_C \end{pmatrix} = \underbrace{\begin{pmatrix} Y_{AB}+Y_{AC} & -Y_{AB} & -Y_{AC} \\ -Y_{AB} & Y_{BC}+Y_{AB} & -Y_{BC} \\ -Y_{AC} & -Y_{BC} & Y_{AC}+Y_{BC} \end{pmatrix}} \times \begin{pmatrix} theta_A \\ theta_B \\ theta_C \end{pmatrix}$$

Note*: the pattern of the Susceptance matrix is that given by Network Theory in that the diagonal term Y_{ii} are the sum of all Y connected to node I and the non-diagonal term Y_{ij} are the negative of the sum of Y directly connecting node i to node j.

The known values are Y_{AB}, Y_{AC} and Y_{BC} (derived from the reciprocals of X_{AB}, X_{AC} and X_{BC}) and P_A, P_B and P_C from the net sum of scaled generation and demand at each node. Hence using matrix inversion you can solve for $theta_A$, $theta_B$ and $theta_C$.